(8 pages) **Reg. No. :** ....

Code No. : 20386 E **Sub. Code : CSMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

Skill Based Subject — VECTOR CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The directional derivative of $\phi(x,y,z) = x^3 + y^3 + z^3$ at the point (1, –1, 2) is ————

(a) $3\bar{i} + 4\bar{j} + 3\bar{k}$ (b) $3\bar{i} + 3\bar{j} + 12\bar{k}$

(c) $3\bar{i} + 3\bar{j} + 3\bar{k}$ (d) $3\bar{i} + 2\bar{j} + 2\bar{k}$

2. The unit vector normal to the surface $\phi = C$ is ————

(a) $\dfrac{\nabla\phi}{|\nabla\phi|}$ (b) $\nabla\phi$

(c) $\nabla^2\phi$ (d) $\dfrac{|\nabla\phi|}{\nabla\phi}$

3. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla.\bar{r} =$ ————

(a) $2x$ (b) $3y$

(c) 3 (d) 4

4. If the vector $(2x,z)\bar{i} + (4x - 11y + 3z)\bar{j} + (3x + mz)\bar{k}$ is solenoidal, then the value of m is ————

(a) 3 (b) 9

(c) 2 (d) 11

5. If $\bar{f} = x^2\bar{i} - xy\bar{j}$ and C is the straight line joining the points (0, 0) and (1, 1), then $\int_C \bar{f}.d\bar{r}$ is ————

(a) 1 (b) 0

(c) –1 (d) 2

6. If $\overline{F} = z(x\overline{i} + y\overline{j} + z\overline{k})$ and C is the straight line joint (0, 0, 0) and (1, 1, 1), then $\int_C \overline{f}.d\overline{r}$ is ————

(a) 0 (b) −1

(c) 1 (d) 2

7. If S is any closed surface enclosing a volume V and $\overline{f} = ax\overline{i} + by\overline{j} + cz\overline{k}$, then $\iint_S \overline{f}.\overline{n}\, dS =$ ————

(a) $(a+b+c)V$ (b) $3V$

(c) $(a+b+c)^3V^3$ (d) 0

8. The value of $\int_0^a\int_0^a\int_0^a x^2 ydzdydx$ is ————

(a) $\frac{a^3}{3}$ (b) $\frac{a^4}{5}$

(c) $\frac{a^5}{4}$ (d) $\frac{a^6}{6}$

9. The value of $\int_C (3x+4y)dx + (2x-3y)dy$, where C is the circle $x^2 + y^2 = 4$ is ————

(a) 4π (b) -8π

(c) 8π (d) 2π

10. The value of $\iint_C \left[(1+y)z\overline{i} + (1+z)x\overline{j} + (1+x)y\overline{k}\right].d\overline{r}$, where C is a closed curve in the plane $x-2y+z=1$ is ————

(a) 2 (b) −1

(c) 0 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) Find the directional derivative of $\phi = x + xy^2 + yz^3$ at (0, 1, 1) in the direction of the vector $2\overline{i} + 2\overline{j} - \overline{k}$.

Or

(b) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $|\overline{r}| = r$, prove that $\nabla\left(\frac{1}{r}\right) = \frac{-\overline{r}}{r^3}$.

12. (a) If $\overline{A} = axy\overline{i} + (x^2 + 2yz)\overline{j} + y^2\overline{k}$ is irrotational, find the value of 'a'.

Or

(b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where 'n' is a constant.

[P.T.O.]

13. (a) Evaluate $\int_C \overline{F}.d\overline{r}$ where $\overline{F} = yz\overline{i} + xz\overline{j} - xy\overline{k}$ and C is the straight line having end points $O(0,0,0)$ and $P(2,4,8)$.

Or

(b) If $\overline{f} = 3xy\overline{i} - y^3\overline{j}$, compute $\int_C \overline{F}.d\overline{r}$ along $y = 2x^2$ from (0, 0) to (1, 2).

14. (a) If $\overline{A} = curl\overline{F}$, compute $\iint_S \overline{A}.\hat{n}\, dS$ for any closed surface S.

Or

(b) Evaluate $\iiint_V \nabla.\overline{F}\, dV$ if $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ and if V is the volume of the region enclosed by the cube $0 \le x, y, z \le 1$.

15. (a) Evaluate $\int_C xydx - x^2dy$ by converting it into a double integral. If is given that the boundary of the region bounded by the line $y = x$ and the parabola $x^2 = y$.

Or

(b) Evaluate by Green's theorem $\int_C e^{-x}(\sin ydx + \cos ydy)$ where C is the rectangle with vertices $(0,0), (\pi,0), (\pi,\pi/2), (0,\pi/2)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) If $\nabla\phi = (y + y^2 + z^2)\overline{i} + (x + z + 2xy)\overline{j} + (y + 2zx)\overline{k}$ and if $\phi(1,,11) = 3$, find ϕ.

Or

(b) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $\overline{r} = r\hat{r}$ show that (i) $\nabla\,(f(r)\overline{r} = rf'(r) + 3f(r)$ (ii) $\nabla \times (f(r)\overline{r}) = \overline{0}$.

17. (a) Find the value of 'm' if $\overline{F} = (6xy + z^3)\overline{i} + (mx^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$ is irrotational. Find also ϕ such that $\overline{F} = \nabla\phi$.

Or

(b) Show that

(i) $(\overline{V}.\nabla)\overline{r} = \overline{V}$

(ii) $(\overline{V} \times \nabla) \times \overline{r} = -2\overline{V}$.

18\. (a) If $\bar{f} = (3x^2 + 6y)\bar{i} - 14yz\bar{j} + 20xz^2\bar{k}$, evaluate $\int_C \bar{F}.d\bar{r}$ where C is (i) a curve whose parametric equations are $x = t, y = t^2, z = t^3$ (ii) straight lines OA, AB, BP where A is $(1, 0, 0)$, B is $(1, 1, 0)$, $O(0, 0, 0)$ and P is $(1,1,1)$.

Or

(b) Evaluate $\iint_S \bar{A}.\bar{n}\, dS$ over the surface S of the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$ if $\bar{A} = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$.

19\. (a) Verify Gauss divergence theorem for $\bar{f} = (x^3 - yz)\bar{i} - 2x^2y\bar{j} + 2\bar{k}$ over the cube bounded by $x = 0, y = 0, z = 0$, $x = a, y = a, z = a$.

Or

(b) Verify Gauss divergence theorem for $\bar{A} = a(x+y)\bar{i} + a(y-x)\bar{j} + z^2\bar{k}$ taken over the region V bounded by the upper hemisphere $x^2 + y^2 + z^2 = a^2$ and the plane $z = 0$.

20\. (a) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region R enclosed by the parabolas $y = x^2$ and $y^2 = x$.

Or

(b) Verify Stoke's theorem for $\bar{F} = y^2\bar{i} + y\bar{j} - xz\bar{k}$ over the upper half of the surface of the sphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.